## Cambridge IGCSE ${ }^{\text {TM }}$



CENTRE NUMBER


CANDIDATE NUMBER

## CAMBRIDGE INTERNATIONAL MATHEMATICS

Paper 5 Investigation (Core)
May/June 2020
1 hour 10 minutes
You must answer on the question paper.
No additional materials are needed.

## INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.


## INFORMATION

- The total mark for this paper is 36 .
- The number of marks for each question or part question is shown in brackets [ ].


## Answer all the questions.

## INVESTIGATION

## DOTTY POLYGONS

This investigation is about the number of dots in shapes that are regular polygons.
For any dotty polygon

- $\quad p$ is the number of sides
- $n$ is the number of dots on one side
- there are the same number of dots on each side.

Example
This is a dotty triangle.


In this dotty triangle, $p=3$ and $n=4$.

1 (a) Look at the numbers of dots in each row of the example.
Complete this sum for the total number of dots in the dotty triangle.

$$
\begin{equation*}
1+2+3+\ldots \ldots . . . . . .= \tag{2}
\end{equation*}
$$

$\qquad$
(b) For a dotty triangle where $n=10$, complete this sum and find the total number of dots.

$$
1+2+3+\ldots \ldots . .+\ldots \ldots . .+\ldots \ldots . .+\ldots \ldots . .+.
$$

$\qquad$ + $\qquad$ $+$ $\qquad$ $=$
(c) Show that $\frac{n^{2}}{2}+\frac{n}{2}$ gives the correct number of dots when $n=10$.

2 The diagram shows the first four dotty triangles. The number of dots added each time is $d$.

$n=1$
$n=2$
$n=3$
$n=4$
$d=1$
$d=2$
$d=3$
$d=4$

So, for dotty triangles, $d=n$.
This diagram shows the first three dotty squares.
$n=1$
$n=2$
$n=3$
$n=4$
$d=1$
$d=3$
$d=5$
$d=7$
(a) Draw the dotty square for $n=4$ in the space above.
(b) (i) Write down the total number of dots in each of the first four dotty squares.
$\qquad$
(ii) Write down an expression, in terms of $n$, for the total number of dots in the $n$th dotty square.
(c) For dotty squares, find a formula for $d$ in terms of $n$.
(d) A formula for $d$, in terms of $p$ (the number of sides) and $n$ is

$$
d=(p-2) n-p+3 .
$$

By substituting appropriate values for $p$, show that this formula gives
(i) the formula for dotty triangles,
(ii) your formula for dotty squares.

3 (a) For dotty pentagons, show that the formula in Question 2(d) becomes $d=3 n-2$.
(b) This diagram shows the first three dotty pentagons.
$d=1$
total $=1$
$d=4$
$d=7$
total $=5$
total $=12$

Dotty pentagons grow along the grey lines.
This diagram shows how to form the first three dotty pentagons.

(i) Use $d=3 n-2$ to find the number of dots that you add to the 3 rd dotty pentagon to make the 4 th dotty pentagon.
$\qquad$
(ii) Complete the diagram to show the 4th and 5th dotty pentagons.
(iii) Complete the final statement.

| 1 st pentagon | +4 dots | $=3$ nd pentagon |
| :--- | :--- | :--- |
| 2 nd pentagon | +7 dots | $=3$ rd pentagon |

........th pentagon +52 dots $=$.......th pentagon

4 (a) This table shows the total number of dots in some dotty polygons.
Use Question 2, Question 3 and any patterns you notice to help you complete this table.

|  |  | Position of dotty polygon in its sequence |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Polygon | $p$ | 1st | 2nd | 3rd | 4th | 5th | $n$th |  |
| Triangle | 3 | 1 | 3 | 6 | 10 |  |  | $\frac{n^{2}}{2}+\frac{n}{2}$ |
| Square | 4 | 1 | 4 | 9 |  |  |  |  |
| Pentagon | 5 | 1 | 5 | 12 |  |  |  |  |
| Hexagon | 6 | 1 | 6 |  |  |  |  |  |

(b)

| The number of dots in a |
| :---: |
| dotty pentagon $\times 3$ |$\quad=\quad$| The number of dots in a |
| :---: |
| dotty triangle |

(i) Give two examples from the table that show this statement is true
(ii)

The number of dots in the 4th $=\quad$ The number of dots in the $k$ th dotty pentagon $\times 3$ dotty triangle

Find the value of $k$.

## BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.

